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Thermalization of Poincaré vacuum state and fermion emission from AdS_3 black holes in bulk-boundary correspondence

Nobuyoshi Ohta and Jian-Ge Zhou

Department of Physics, Osaka University
Toyonaka, Osaka 560-0043, Japan
E-mail: ohta@phys.wani.osaka-u.ac.jp, jgzhou@phys.wani.osaka-u.ac.jp

Abstract: The greybody factors for spin $\frac{1}{2}$ particles in the BTZ black holes are discussed from 2D CFT in bulk-boundary correspondence. It is found that the initial state of spin $\frac{1}{2}$ particle in the BTZ black holes can be described by the Poincaré vacuum state in boundary 2D CFT, and the nonlinear coordinate transformation causes the thermalization of the Poincaré vacuum state. For special case, our results for the greybody factors agree with the semiclassical calculation.

Keywords: Black Holes in String Theory, Conformal and W Symmetry, Models of Quantum Gravity, Black Holes.

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In recent years there has been great progress in our understanding of black hole physics from string and conformal field theories (CFT). For reviews on this subject, see refs. [1]. Through these studies, it has been gradually realized that some 5D and 4D black holes contain BTZ black holes [2] in the near-horizon region [3, 4], and higher-dimensional black hole physics is essentially encoded in that of BTZ black holes. In fact it has been shown in ref. [5] that the Bekenstein-Hawking entropy for 5D and 4D black holes can be related to the entropy of BTZ black holes by making use of U-duality.

Though much work on the absorption and Hawking radiation in 5D and 4D black holes has also been made in the semiclassical analysis, D-brane picture, effective string model and effective 2D CFT in refs. [6]-[10], only recently one began to recognize that not only the Bekenstein-Hawking entropy but also the greybody factors in 5D and 4D black holes can be understood as effectively coming from the near-horizon BTZ geometry [11, 12, 13]. Since the recently discovered AdS/CFT correspondence [3, 14, 15] might play an important role in the fundamental quantum theory for the black holes via the near-horizon BTZ black holes, it is natural to expect that the greybody factors in the black holes should be elucidated in the context of the AdS/CFT correspondence.

Indeed the greybody factors for scalar fields have been derived by using the near-horizon BTZ (AdS_3) geometry and AdS/CFT correspondence [11, 12]. In ref. [12], the initial state of scalar particles in BTZ black holes has been described by a Poincaré vacuum state in the boundary 2D CFT. The nonlinear coordinate transformation between the Poincaré coordinates (w_+, w_-) and the BTZ coordinates (u_+, u_-) induces a mapping of the operator $\mathcal{O}(w_+, w_-)$ to $\mathcal{O}(u_+, u_-)$ by Bogoliubov transformation, and the operator $\mathcal{O}(u_+, u_-)$ sees the Poincaré vacuum state as an excited mixed state, that is, they see the Poincaré vacuum state as thermal bath of excitations in BTZ modes [12, 16, 17]. The usual procedure to thermally average the initial state of scalar particles in the calculation of greybody factors is just to measure the Poincaré vacuum state by the operator $\mathcal{O}(u_+, u_-)$ in the BTZ coordinates.

In this paper, we discuss the greybody factors for spin $\frac{1}{2}$ particle in the BTZ black holes from AdS/CFT correspondence. Though there have been many studies of correlation functions in the boundary theory from the bulk-boundary correspondence [18, 19, 20], these calculations have mostly been performed in the Poincaré coordinates. Only a few papers discussed the two-point correlation functions for scalar fields in the BTZ coordinates [12, 17].

To describe fermion emission from the BTZ black holes in the spirit of AdS/CFT correspondence, we need to calculate two-point correlation functions for spinor fields in the BTZ coordinates (including all coefficients in the calculation). As we know, the bulk-boundary Green functions for scalar fields in the BTZ coordinates only depend on the differences of the coordinates $(\Delta u_+, \Delta u_-)$ [12, 17], and so it is manifestly invariant under the translations in the boundary BTZ coordinates. However, for spinor case, though the bulk-boundary Green function in Poincaré coordinates (w_+, w_-) is a function of Δw_+ and Δw_-, its form in the BTZ coordinates (u_+, u_-) not only depends on Δu_+ and Δu_-, but also on $(u_+, u_-; u'_+, u'_-)$. The reason is that when we construct bulk-boundary Green functions in the Poincaré coordinates, we apply an element of the $O(3,1)$ isometry group of AdS_3 to move the singularity from $y = \infty$ to an arbitrary point on the boundary which has to be accompanied by a compensating local Lorentz transformation for spinor fields to preserve the gauge fixing on the dreibein [18], and this local Lorentz transformation in the bulk breaks the manifest invariance of the bulk-boundary Green functions under the translations in the boundary BTZ coordinates. Due to this special behavior of the bulk-boundary Green functions in the BTZ coordinates, it is highly nontrivial to check whether the two-point correlation functions for spinor fields in the BTZ coordinates take the expected form with translational invariance [9, 12]. Remarkably we find that this is indeed true and that the greybody factors for spinor fields in the BTZ black holes obtained from AdS/CFT correspondence agree with the known results for a special case including the coefficients.

Let us first consider two-point correlation functions of the operators coupling to the boundary values of spinor fields in the Poincaré coordinates. The AdS_3 metric in the Poincaré coordinates is

$$ds^2 = \frac{l^2}{y^2}(dy^2 + dw_+ dw_-)\,. \tag{1}$$

For simplicity, we choose the radius of our AdS_3 space $l = 1$ in the following discussions.

The free spinor action on AdS_3 is[1]

$$S_0 = \frac{1}{2}\int dy dw_+ dw_- \sqrt{g}\,\bar{\Psi}(\not{D} - m)\Psi\,. \tag{2}$$

For spinor fields, the action (2) vanishes for the field configuration that satisfies the equation of motion. The free action (2) should be supplemented by a boundary term [18], which can be induced from the Hamiltonian version of AdS/CFT correspondence [21]

$$S_1 = \lim_{y\to 0}\frac{1}{4}\int dw_+ dw_- \sqrt{g_0}\,\bar{\Psi}\Psi\,, \tag{3}$$

where g_0 is the determinant of the induced metric y^{-4} [18]. It has been shown that the theory thus defined is equivalent to CFT on the two-dimensional boundary even though the bulk action in AdS_3 looks not conformally invariant in three dimensions [18].

[1]We take $\Gamma^y = \sigma^3, \Gamma^1 = \sigma^1, \Gamma^2 = \sigma^2$ and $w^\pm = x^1 \mp i x^2$ in Euclidean case, $\Gamma^0 = -i\Gamma^2, w^\pm = x^1 \pm t$ in Minkowski case, and $\Gamma^\pm = (\Gamma^1 \pm \Gamma^0)/2$. Note that $dtdx = \frac{1}{2}dw_+ dw_-$.

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The bulk field $\Psi(y, w_+, w_-)$ and $\bar{\Psi}(y, w_+, w_-)$ can be obtained from the boundary value $\psi(w_+, w_-)$ and $\bar{\psi}(w_+, w_-)$ by [18]

$$\begin{aligned}
\Psi\left(y, w_+, w_-\right) = & \frac{m+\frac{1}{2}}{2\pi} \int dw'_+ dw'_- \left[y\Gamma^y + \left(w_+ - w'_+\right)\Gamma^- + \left(w_- - w'_-\right)\Gamma^+\right] \times \\
& \times \left[y^2 + \left(w_+ - w'_+\right)\left(w_- - w'_-\right)\right]^{-3/2 + m\Gamma^y} y^{1-m\Gamma^y} \psi\left(w'_+, w'_-\right), \\
\bar{\Psi}\left(y, w_+, w_-\right) = & \frac{m+\frac{1}{2}}{2\pi} \int dw'_+ dw'_- \bar{\psi}\left(w'_+, w'_-\right) y^{1+m\Gamma^y} \times \\
& \times \left[y^2 + \left(w_+ - w'_+\right)\left(w_- - w'_-\right)\right]^{-3/2 - m\Gamma^y} \times \\
& \times \left[y\Gamma^y + \left(w_+ - w'_+\right)\Gamma^- + \left(w_- - w'_-\right)\Gamma^+\right].
\end{aligned} \tag{4}$$

If we take mass m positive, we have to choose $\Gamma^y\psi(w_+, w_-) = -\psi(w_+, w_-)$ and $\bar{\psi}(w_+, w_-)\Gamma^y = \bar{\psi}(w_+, w_-)$ for the consistency of the theory [18], which means that $\psi(w_+, w_-)$ can be written as

$$\psi\left(w_+, w_-\right) = \begin{pmatrix} 0 \\ \psi_0\left(w_+, w_-\right) \end{pmatrix}. \tag{5}$$

Then eq. (4) is recast into

$$\begin{aligned}
\Psi\left(y, w_+, w_-\right) &= \int dw'_+ dw'_- K_P\left(y, w_+, w_-; w'_+, w'_-\right) \psi_0\left(w'_+, w'_-\right), \\
\bar{\Psi}\left(y, w_+, w_-\right) &= \int dw'_+ dw'_- \psi_0^\dagger\left(w'_+, w'_-\right) \tilde{K}_P\left(y, w_+, w_-; w'_+, w'_-\right),
\end{aligned} \tag{6}$$

with the normalized bulk-boundary Green functions in the Poincaré coordinates given by

$$\begin{aligned}
K_P\left(y, w_+ - w'_+, w_- - w'_-\right) = & \frac{m+\frac{1}{2}}{2\pi} y^{m+1} \left[y^2 + \left(w_+ - w'_+\right)\left(w_- - w'_-\right)\right]^{-m-3/2} \times \\
& \times \begin{pmatrix} w_+ - w'_+ \\ -y \end{pmatrix}, \\
\tilde{K}_P\left(y, w_+ - w'_+, w_- - w'_-\right) = & -\frac{m+\frac{1}{2}}{2\pi} y^{m+1} \left[y^2 + \left(w_+ - w'_+\right)\left(w_- - w'_-\right)\right]^{-m-3/2} \times \\
& \times \left(y, w_+ - w'_+\right),
\end{aligned} \tag{7}$$

where K_P and $\tilde{K}_P$ are manifestly invariant under the translations in the boundary Poincaré coordinates (w_+, w_-).

The coupling between the operators and the boundary values of spinor fields takes the form

$$S(\bar{\psi}, \psi) = \frac{1}{2} \int dw_+ dw_- \left(\bar{\mathcal{O}}\psi + \bar{\psi}\mathcal{O}\right)\left(w_+, w_-\right). \tag{8}$$

Owing to eq. (5), without any loss of generality, we can choose

$$\mathcal{O}(w_+, w_-) = \begin{pmatrix} \mathcal{O}_0(w_+, w_-) \\ 0 \end{pmatrix} . \tag{9}$$

The coupling (8) is then reduced to

$$S = \frac{1}{2} \int dw_+ dw_- \left(-\mathcal{O}_0^\dagger \psi_0 + \psi_0^\dagger \mathcal{O}_0 \right) (w_+, w_-) . \tag{10}$$

According to AdS/CFT correspondence, the relation between string theory in the bulk and field theory on the boundary is [15]

$$e^{-S_1(\bar{\Psi}, \Psi)} = \langle e^{S(\bar{\psi}, \psi)} \rangle_{CFT} . \tag{11}$$

The two-point correlation function for spinor fields in the Poincaré coordinates is [18]

$$\left\langle \mathcal{O}_0^\dagger (w_+, w_-) \, \mathcal{O}_0 \left(w'_+, w'_- \right) \right\rangle = \frac{m + \frac{1}{2}}{\pi} \frac{1}{(w_+ - w'_+)^{2h_+} (w_- - w'_-)^{2h_-}} , \tag{12}$$

with

$$h_+ = h - \frac{1}{4} , \qquad h_- = h + \frac{1}{4} , \qquad h = \frac{m+1}{2} , \qquad h_- - h_+ = \frac{1}{2} , \tag{13}$$

which shows that the conformal dimensions for $\mathcal{O}_0^\dagger(w_+, w_-)$ and $\mathcal{O}_0(w_+, w_-)$ are (h_+, h_-) when we take m to be positive.

From the conformal invariance of the action (10), we know that the conformal dimensions for $\psi_0^\dagger(w_+, w_-)$ and $\psi_0(w_+, w_-)$ are $(1 - h_+, 1 - h_-)$.

Now we turn to the two-point correlation functions for spinor fields in the BTZ coordinates. The metric of the BTZ black holes is [2]

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2 \left(d\phi - \frac{r_+ r_-}{r^2} dt \right)^2 , \tag{14}$$

with periodic identification $\phi \sim \phi + 2\pi$, where we have chosen $l = 1$. The mass and angular momentum are defined as

$$M = r_+^2 + r_-^2 , \qquad J = 2 r_+ r_- . \tag{15}$$

It has been shown that the metric of BTZ black holes can be transformed to that of AdS_3 locally by the transformation which in the region $r >> r_\pm$ takes the form [16, 17]

$$w_\pm = e^{2\pi T_\pm u_\pm} , \tag{16}$$

$$y = \left(\frac{r_+^2 - r_-^2}{r^2} \right)^{\frac{1}{2}} e^{\pi (T_+ u_+ + T_- u_-)} , \tag{17}$$

with

$$T_{\pm} = \frac{r_{+} \mp r_{-}}{2\pi}, \qquad u_{\pm} = \phi \pm t. \tag{18}$$

The boundary fields $\psi_0^\dagger(w_+, w_-)$ and $\psi_0(w_+, w_-)$ have conformal dimensions $(1-h_+, 1-h_-)$, and so they transform as

$$\begin{aligned}
\psi_0\left(w_+, w_-\right) &= \left(\frac{dw_+}{du_+}\right)^{h_+-1}\left(\frac{dw_-}{du_-}\right)^{h_--1}\psi_0\left(u_+, u_-\right),\\
\psi_0^\dagger\left(w_+, w_-\right) &= \left(\frac{dw_+}{du_+}\right)^{h_+-1}\left(\frac{dw_-}{du_-}\right)^{h_--1}\psi_0^\dagger\left(u_+, u_-\right),
\end{aligned} \tag{19}$$

under the transformation (16) and (17).

Combining eqs. (6), (7), (16), (17) and (19), we find that the relation between the bulk and boundary fields in the BTZ coordinates is changed into

$$\begin{aligned}
\Psi\left(r, u_+, u_-\right) &= \int du'_+ du'_- K_B\left(r, u_+, u_-; u'_+, u'_-\right)\psi_0\left(u'_+, u'_-\right),\\
\bar{\Psi}(r, u_+, u_-) &= \int du'_+ du'_- \psi_0^\dagger\left(u'_+, u'_-\right)\tilde{K}_B\left(r, u_+, u_-; u'_+, u'_-\right),
\end{aligned} \tag{20}$$

with

$$\begin{aligned}
K_B\left(r, u_+, u'_+; u_-, u'_-\right) &= \frac{2h-\frac{1}{2}}{2\pi}\left(2\pi T_+\right)^{h_+}\left(2\pi T_-\right)^{h_-} e^{\frac{\pi}{2}(T_+u_+ - T_-u_-)}\left(\frac{r_+^2 - r_-^2}{r^2}\right)^{h}\times\\
&\times\left[\frac{r_+^2 - r_-^2}{r^2}e^{\pi(T_+\Delta u_+ + T_-\Delta u_-)} + 4\sinh\left(\pi T_+\Delta u_+\right)\sinh\left(\pi T_-\Delta u_-\right)\right]^{-\left(2h+\frac{1}{2}\right)}\times\\
&\times\begin{pmatrix} 2\sinh(\pi T_+\Delta u_+) \\ -\sqrt{\frac{r_+^2-r_-^2}{r^2}}e^{\pi(-T_+u'_+ + T_-u_-)}\end{pmatrix},\\
\tilde{K}_B\left(r, u_+, u'_+; u_-, u'_-\right) &= \frac{(2h-\frac{1}{2})}{2\pi}(2\pi T_+)^{h_+}(2\pi T_-)^{h_-} e^{\frac{\pi}{2}(T_+u_+ - T_-u_-)}\left(\frac{r_+^2 - r_-^2}{r^2}\right)^{h}\times\\
&\times\left[\frac{r_+^2 - r_-^2}{r^2}e^{\pi(T_+\Delta u_+ + T_-\Delta u_-)} + 4\sinh(\pi T_+\Delta u_+)\sinh(\pi T_-\Delta u_-)\right]^{-(2h+\frac{1}{2})}\times\\
&\times\left(-\sqrt{\frac{r_+^2 - r_-^2}{r^2}}e^{\pi(-T_+u'_+ + T_-u_-)},\ -2\sinh(\pi T_+\Delta u_+)\right),
\end{aligned} \tag{21}$$

and

$$\Delta u_+ = u_+ - u'_+, \qquad \Delta u_- = u_- - u'_-. \tag{22}$$

Note that in contrast to $K_P(y, \Delta w_+, \Delta w_-)$ and $\tilde{K}_P(y, \Delta w_+, \Delta w_-)$, K_B and $\tilde{K}_B$ are not manifestly invariant under the translations in the boundary coordinates (u_+, u_-). This is because when we apply an element of the $O(3,1)$ isometry group of AdS_3 to construct the solution (4), we have to compensate local Lorentz transformation for spinor fields to preserve the gauge choice [18], and this local Lorentz transformation in the bulk breaks the manifest invariance of the bulk-boundary Green functions under translations in the boundary BTZ coordinates. Nevertheless, we will show that these Green functions give translationally invariant two-point correlation functions in the boundary coordinates.

In the BTZ coordinates, the boundary action S_1 in (3) turns into

$$S_1 = \lim_{r\to\infty} \frac{1}{4} \int du_+ du_- \left(\frac{r^2}{r_+^2 - r_-^2}\right) \bar{\Psi}(r, u_+, u_-)\Psi(r, u_+, u_-) \,. \tag{23}$$

Inserting eqs. (20) and (21) into (23), we have[2]

$$\begin{aligned} S_1 = \frac{h_+}{8\pi} \int du_+ du_- du'_+ du'_- \psi_0^\dagger \left(u'_+, u'_-\right) \left(\frac{\pi T_+}{\sinh(\pi T_+ \Delta u_+)}\right)^{2h_+} \times \\ \times \left(\frac{\pi T_-}{\sinh(\pi T_- \Delta u_-)}\right)^{2h_-} \psi_0(u_+, u_-) \,. \end{aligned} \tag{24}$$

In the BTZ coordinates, the boundary action (10) for the operators $\mathcal{O}_0^\dagger(w_+, w_-)$, $\mathcal{O}_0(w_+, w_-)$ and the boundary values of spinor fields can be written as

$$S = \frac{1}{2} \int du_+ du_- (-\mathcal{O}_0^\dagger \psi_0 + \psi_0^\dagger \mathcal{O}_0)(u_+, u_-) \,. \tag{25}$$

By using eq. (11), one finds

$$\begin{aligned} G(t, \phi) &= \langle \mathcal{O}_0^\dagger(u_+, u_-) \mathcal{O}_0(0, 0) \rangle \\ &= \frac{h_+}{2\pi} \left(\frac{\pi T_+}{\sinh \pi T_+ u_+}\right)^{2h_+} \left(\frac{\pi T_-}{\sinh \pi T_- u_-}\right)^{2h_-} \,, \end{aligned} \tag{26}$$

which indicates that the operators $\mathcal{O}_0^\dagger(u_+, u_-)$ and $\mathcal{O}_0(u_+, u_-)$ see the Poincaré vacuum state as an excited mixed state, that is, they see the Poincaré vacuum state as thermal bath of excitations in BTZ modes [12, 16, 17].

Before proceeding, it would be appropriate to compare the above method to derive (26) with Unruh's calculation [22]. Here the equation of motion in the background of three-dimensional AdS space is solved and the bulk field is expressed by the boundary value of the corresponding field with the help of the normalized bulk-boundary Green function. By using the normalized Green function in the BTZ

[2] In deriving (24), we have exploited the formula

$$\lim_{r\to\infty} \left(\frac{r_+^2 - r_-^2}{r^2}\right)^{2h - \frac{1}{2}} \left[\frac{r_+^2 - r_-^2}{r^2} e^{\pi(T_+ \Delta u_+ + T_- \Delta u_-)} + 4\sinh(\pi T_+ \Delta u_+)\sinh(\pi T_- \Delta u_-)\right]^{-(2h+\frac{1}{2})} = \\ = \frac{\pi}{2h_+(2\pi T_+)(2\pi T_-)} \delta(\Delta u_+)\delta(\Delta u_-) \,.$$

coordinates, the conformal dimensions for boundary fields and eqs. (16) and (17), we can obtain eq. (26). In Unruh's calculation for the fermion [22], on the other hand, the equation of motion in two-dimensional collapsing-shell metric was solved and a conformal transformation similar to (16) was exploited but not (17), since the equation of motion was analysed only in two dimensions. Thus the above method in the context of AdS/CFT correspondence is essentially different from Unruh's calculation, but relies on Witten's conjecture [15].

It is known that the normalization factor cannot be fixed in the effective CFT without recourse to string theory [9]. However with the help of the normalized bulk-boundary Green function we can determine the normalization factor for spin $\frac{1}{2}$ particles. Also from the bulk-boundary correspondence, the dependence of conformal dimensions on the AdS_3 mass m can be read off easily.

Because of the periodic identification of the coordinate ϕ, the two-point correlation function $G(t,\phi)$ should be modified as [12, 17]

$$\begin{aligned} G_T(t,\phi) &= \langle \mathcal{O}_0^\dagger(u_+,u_-)\mathcal{O}_0(0,0)\rangle = \\ &= \frac{h_+}{2\pi}\sum_{n=-\infty}^{\infty}\left(\frac{\pi T_+}{\sinh \pi T_+(\phi+t+2n\pi)}\right)^{2h_+}\left(\frac{\pi T_-}{\sinh \pi T_-(\phi-t+2n\pi)}\right)^{2h_-}, \end{aligned} \tag{27}$$

where the terms for $n \neq 0$ come from the twisted sectors of the operators $\mathcal{O}_0^\dagger(u_+,u_-)$ and $\mathcal{O}_0(u_+,u_-)$ in the orbifold procedure $u_\pm \sim u_\pm + 2n\pi$ for the BTZ black holes [16].

The greybody factors for spinor fields in the BTZ black holes are given by [9, 11]

$$\begin{aligned} \sigma_{abs} &= \frac{2\pi}{\mathcal{F}}\int dt\int_0^{2\pi} d\phi e^{ip\cdot x}[G_T(t-i\epsilon,\phi)-G_T(t+i\epsilon,\phi)] \\ &= \frac{2\pi}{\mathcal{F}}\int dt\int_{-\infty}^{\infty} d\phi e^{ip\cdot x}[G(t-i\epsilon,\phi)-G(t+i\epsilon,\phi)] \\ &= \frac{h_+}{\mathcal{F}}\frac{(2\pi T_+)^{2h_+-1}(2\pi T_-)^{2h_--1}}{\Gamma(2h_+)\Gamma(2h_-)}\cosh\left(\frac{\omega}{2T_H}\right)\times \\ &\quad \times\left|\Gamma\left(h_+ + i\frac{\omega}{4\pi T_+}\right)\Gamma\left(h_- + i\frac{\omega}{4\pi T_-}\right)\right|^2, \end{aligned} \tag{28}$$

where $\mathcal{F}$ is the incident fermion flux and we have assumed the scaling dimension for spinor fields is half an odd integer, which is true when we consider the mass term m induced from the Kaluza-Klein reduction from $\mathrm{AdS}_3 \times S^3 \times M_4$ [23]. The Hawking temperature T_H is defined by

$$\frac{2}{T_H} = \frac{1}{T_+} + \frac{1}{T_-}. \tag{29}$$

The above calculation for greybody factors has been performed in the 2D CFT on the two-dimensional boundary, where the metric is given as $ds^2 = dw_+dw_-$ in the boundary Poincaré coordinates (w_+,w_-) [18]. Thus we can use eq. (28) to derive greybody factors from the point of view of 2D CFT.

Usually the greybody factors describe scattering of asymptotic states from asymptotically flat black holes. Since AdS spacetimes do not have asymptotic states in the same sense as in asymptotically flat spacetimes, we should explain what "greybody factors for BTZ black holes" mean. In the limit of large number N of D-branes, the geometries of the 5D and 4D black holes are BTZ $\times S^3 \times M_4$ and BTZ $\times S^2 \times M_5$, respectively [16, 11], but they are asymptotically flat. For low-energy emission, the greybody factors in higher-dimensional black holes computed in gravity for asymptotically flat black holes are related to the two-point correlation functions obtained in large N D-brane gauge theories [6]. In the context of AdS/CFT correspondence, these two-point functions can in turn be computed from semiclassical gravity inside the throat region which becomes BTZ black hole in a suitable limit. Thus we can do a classical gravity calculation to compute the large N gauge theory two-point correlation functions which give the greybody factors for asymptotically flat black holes. All of this shows that the greybody factors in higher-dimensional black holes have their origin in BTZ black holes [11, 12, 13]. The boundary dynamics of BTZ black holes, which is controlled by 2D CFT, looks like hologram and contains the essential informations of higher-dimensional black holes [12].

On the other hand, in asymptotically flat spacetime, the asymptotic observer measures a decay rate which is modified by the greybody factor of the black hole. To define the Hawking emission rate for the BTZ black hole in the gravity calculations, we do not take an asymptotic observer, but an observer stationed at $r \sim l >> r_+$, which means that we take the incoming flux in the region $r \sim l >> r_+$ as the incident flux on the black hole [24, 25]. The reason for this choice is that in curved spacetime, an observer measures a thermal spectrum depending on his/her local temperature $T_H/\sqrt{g_{00}}$, and for asymptotically flat spacetime $\sqrt{g_{00}} \to 1$ as $r \to \infty$. For BTZ black holes with $r_+ << l$, we see that $\sqrt{g_{00}} \to 1$ when $r \sim l$, so the observer in BTZ geometry measures a local temperature equal to the Hawking temperature at this position [25].

To compare the above result with that from the semiclassical calculation, we consider $(h_+, h_-) = (\frac{1}{2}, 1)$ case. According to the definition for $\mathcal{F}$ in refs. [9, 25], we can choose $\mathcal{F} = 1$. We find from eq. (28)

$$\sigma_{abs}^{(\frac{1}{2},1)} = \frac{\pi^2\omega}{4} \frac{\cosh\left(\frac{\omega}{2T_H}\right)}{\cosh\left(\frac{\omega}{4T_+}\right)\sinh\left(\frac{\omega}{4T_-}\right)}. \tag{30}$$

The emission rate of the fermions is given by the product with thermal distribution:

$$\Gamma^{(\frac{1}{2},1)} = \frac{\pi^2\omega}{2} \frac{d^2k}{\left[\exp\left(\frac{\omega}{2T_+}\right)+1\right]\left[\exp\left(\frac{\omega}{2T_-}\right)-1\right]}, \tag{31}$$

which agrees with the semiclassical gravity calculations in ref. [25] in near-extremal limit ($T_- >> T_+$) and for energies small in comparison with the size of the black holes (see eq. (34) there).[3]

The agreement of greybody factors for spinor fields in the BTZ black holes obtained from AdS/CFT correspondence with that from semiclassical gravity calculations [25] supports the identification that the initial state of the particle in the BTZ black holes can be described by the Poincaré vacuum state in the boundary 2D CFT [12], and the nonlinear coordinate transformation (16) and (17) causes the thermalization of the Poincaré vacuum state which holds valid also for the spinor case. We believe that such an identification should also work for spin $\frac{3}{2}$ Rarita-Schwinger fields in BTZ black holes. However, in the above discussions, we have only considered free fields. It would be interesting to check whether such an identification works for interacting theories.

In order to compare the fermion emission from black holes with three charges (Q_1, Q_5, n) in five-dimensional $N = 8$ supergravity between the above AdS/CFT approach and semiclassical analysis [10], we need to calculate two-point correlation functions for spin $\frac{1}{2}$ particle in the near-horizon geometry of 5D black holes. By analogy with the scalar case, we expect that it has the form [12, 20]

$$\left\langle \mathcal{O}_0^\dagger(u_+, u_-)\mathcal{O}_0(0,0)\right\rangle = \eta_{5D}\frac{h_+}{2\pi}\left(\frac{\pi T_+}{\sinh \pi T_+ u_+}\right)^{2h_+}\left(\frac{\pi T_-}{\sinh \pi T_- u_-}\right)^{2h_-}. \tag{32}$$

It is nice to see how to determine the coefficient η_{5D} for the spinor field precisely.

A recent work [26] suggested that in the very near horizon limit the above boundary 2D CFT is transformed into discrete light-cone quantization of a CFT which has a connection with matrix model [27]. It would be interesting to see how the above fermion emission from BTZ black holes in AdS/CFT correspondence is related to that in the context of matrix black holes [28]. Work along this line is under investigation.

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